Outline

- methods
  - definition of reduction algorithm
  - implementation of reduction algorithm
    - » full matrices & full pivoting
    - » partial pivoting & sparse matrices
- results
  - progress of sparse mode by long doubles
  - analysis of irregular digital filters
  - accuracy check comparing MATLAB and CIA analyses
  - estimating a frequency of microwave oscillator

1. Poles-Zeros Analysis

Algorithm

$$(o\boldsymbol{P}+\boldsymbol{Q})\boldsymbol{X}=\boldsymbol{Y},$$

where o denotes operator s for analog circuits and z^{-1} operator for digital circuits, $\boldsymbol{X}$ is a vector of s images of analog circuit variables or z^{-1} images of digital circuit variables, and $\boldsymbol{Y}$ is a source vector.

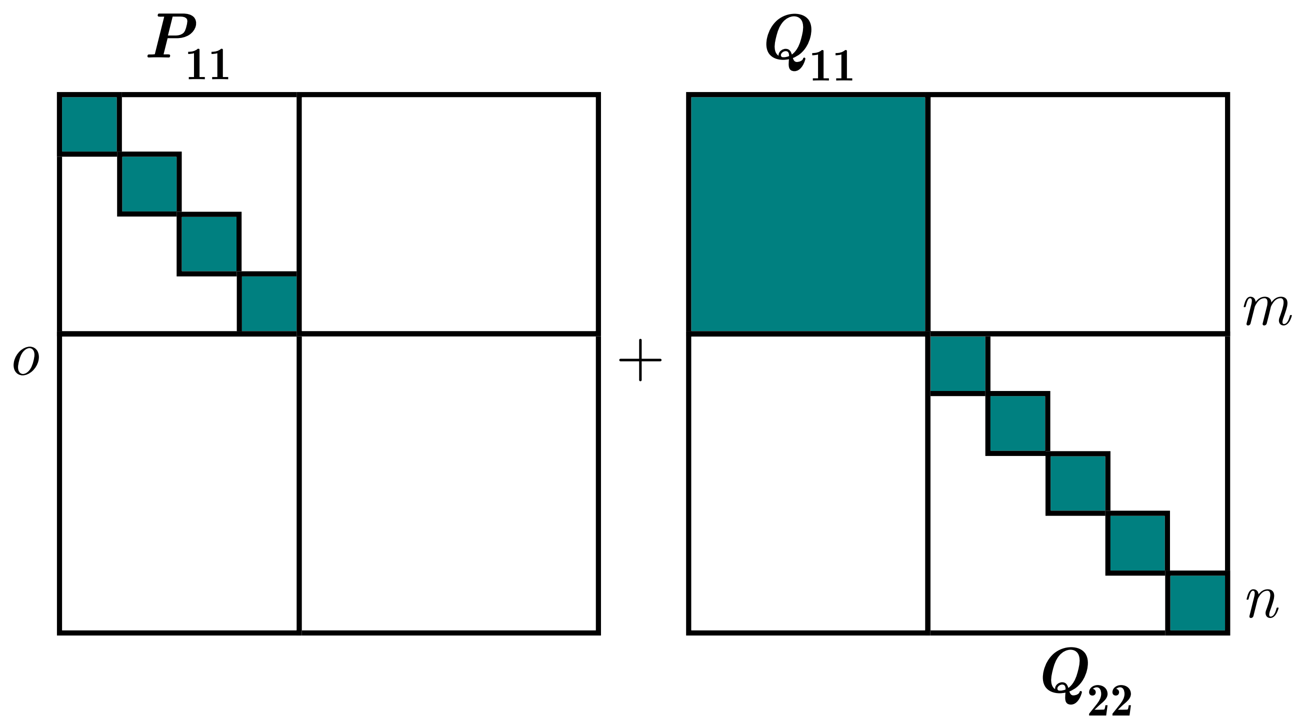
Cramer rule:

$$\det(o\boldsymbol{P}+\boldsymbol{Q})=0 \text{ for poles,}$$

$$\det\left(o\boldsymbol{P}^{\boldsymbol{0j}}+\boldsymbol{Q}^{\boldsymbol{ij}}\right)=0 \text{ for zeros,}$$

$\boldsymbol{P}^{\boldsymbol{0j}}$ arises from the origin $\boldsymbol{P}$ by clearing its j column,

$\boldsymbol{Q}^{\boldsymbol{ij}}$ arises from the origin $\boldsymbol{Q}$ by replacing its j column by cleared $\boldsymbol{Y}$ with the exception of the i exciting source.



the determinant can now be calculated in the classical way:

$$\det(o\boldsymbol{P}+\boldsymbol{Q}) = (-1)^{n_{\text{exch}}} \prod_{i=1}^{m} P_{11_{ii}} \prod_{i=m+1}^{n} Q_{22_{ii}} \det\left(o\mathbf{1}+\boldsymbol{P}_{\mathbf{11}}^{-1}\boldsymbol{Q}_{\mathbf{11}}\right)$$

Implementation

CIA program has implemented two algorithms for reduction: the first operates with full matrix technique and *full pivoting* and is appropriate for accurate computations, the second with *sparse matrix technique* and partial pivoting and is appropriate for analyses of huge circuits.

- **case of the full pivoting:**

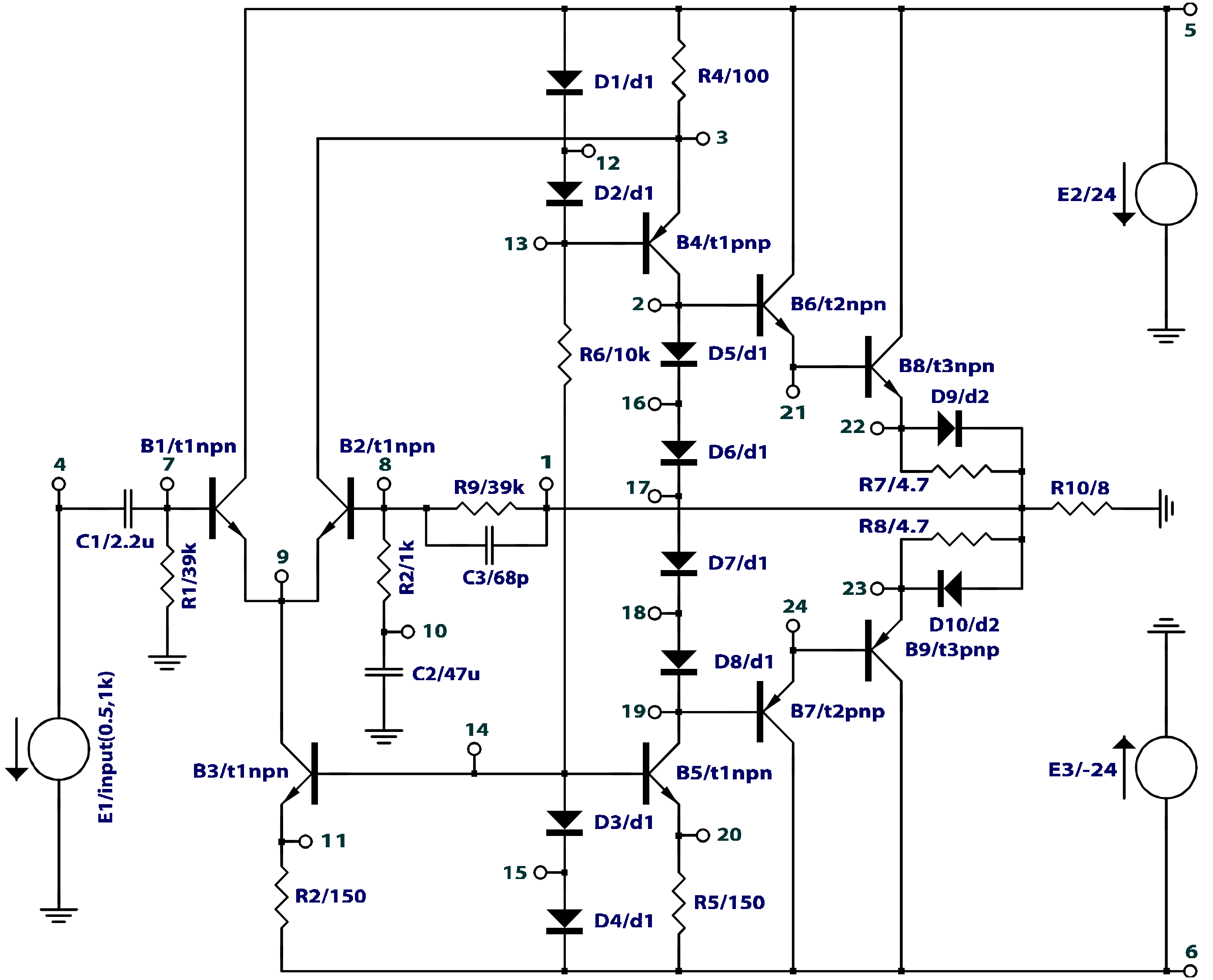
$$P_{kk} \leftarrow \max_{i=k,j=k}^{i=n,j=n} \left| P_{ij} \right|, \, k \in \{1, \ldots, n\}$$

- **case of the partial pivoting:**

$$P_{kk} \leftarrow \max_{i=k}^{n} \left| P_{ik} \right|, \, k \in \{1, \ldots, n\}$$

2. Examples Demonstrating a Progress

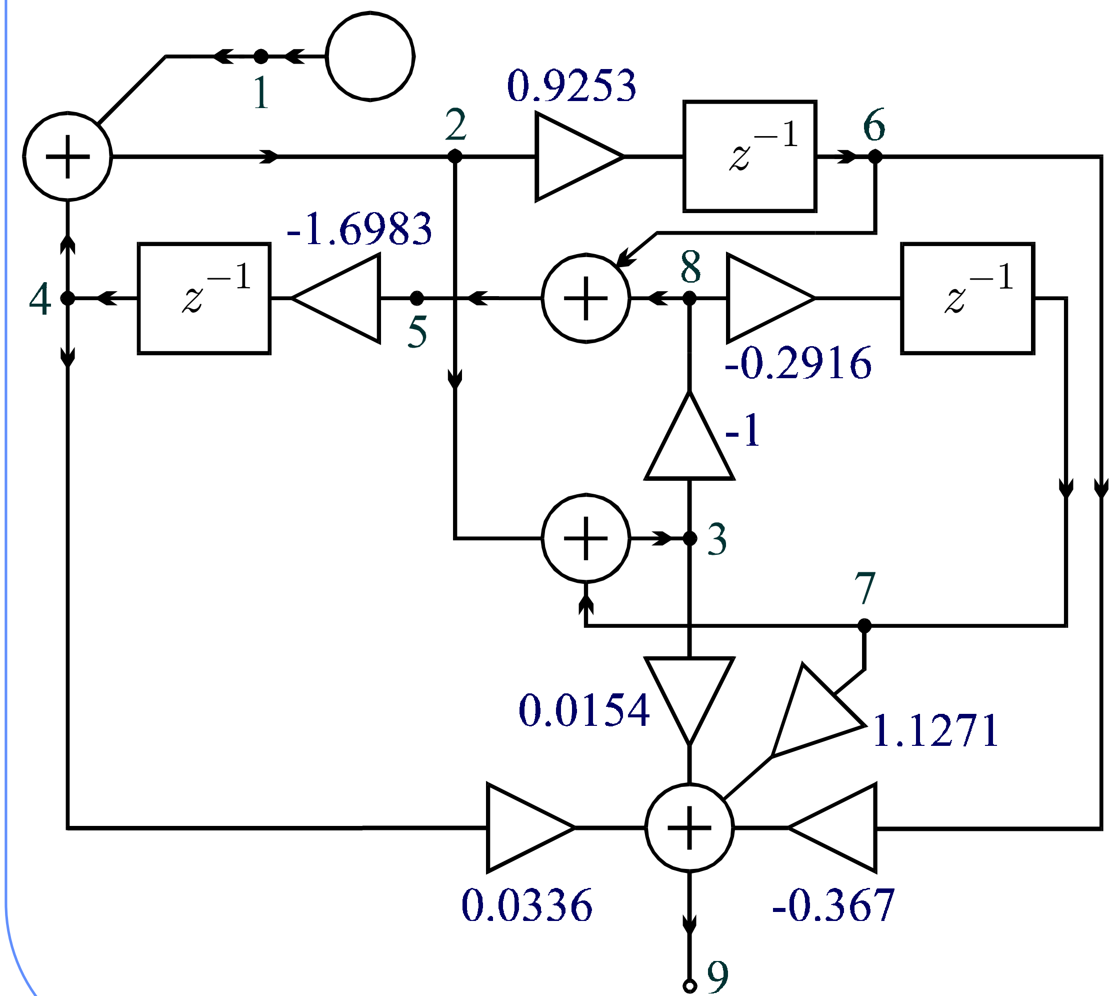
Power Operational Amplifier



Accuracy comparison of sparse double and sparse long double algorithms (ε optimized).

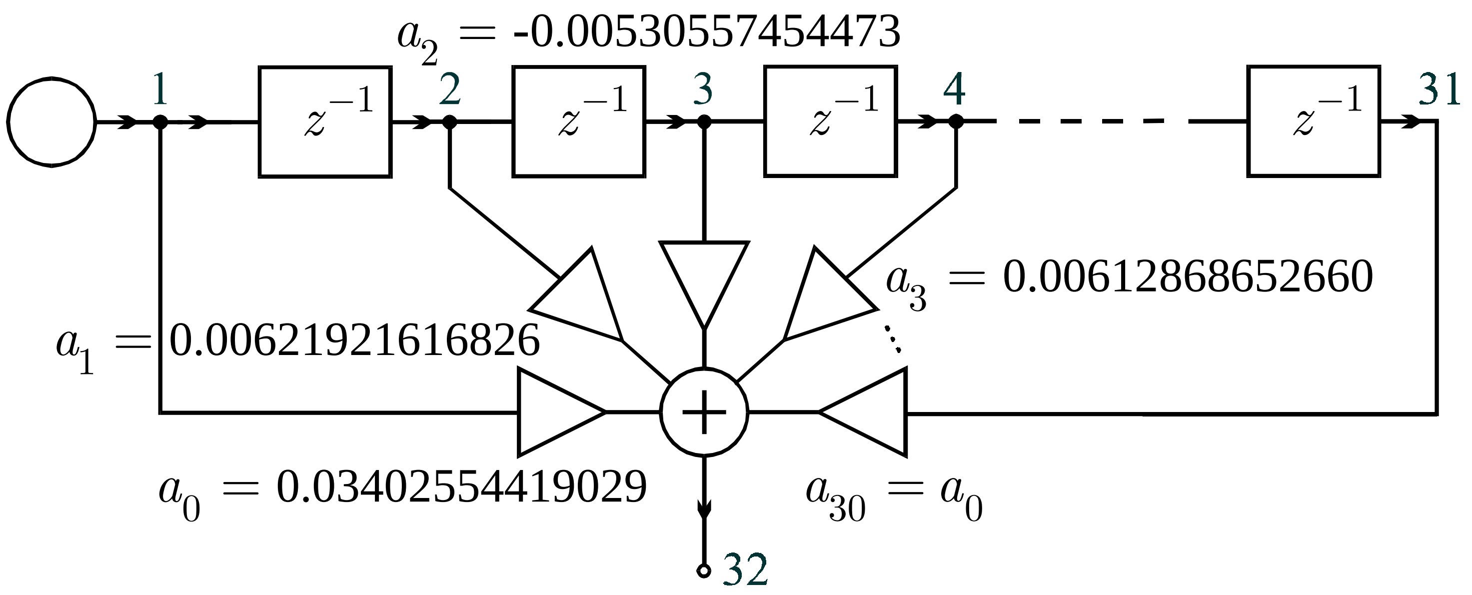
$\varepsilon_{eigen}=5*10^{-15}$, $\varepsilon_{round}=10^{-19}$	full-matrix algorithm	sparse double algor.	sparse long double algor.
smallest pole (by magnitude)	-1.76518 Hz	-1.76518 Hz	-1.76518 Hz
biggest pole (by magnitude)	$-7.31331*10^{10}$ Hz	$-7.31330*10^{10}$ Hz	$-7.31330*10^{10}$ Hz
smallest zero (by magnitude)	-0.0847747 Hz	-0.0847825 Hz	-0.0847825 Hz
biggest zero (by magnitude)	$-6.99856*10^{10}$ Hz	$-7.04891*10^{10}$ Hz	$-6.99278*10^{10}$ Hz
constant of transfer function	0.988898	1.04496	0.989439

Irregular 3rd-order digital filter



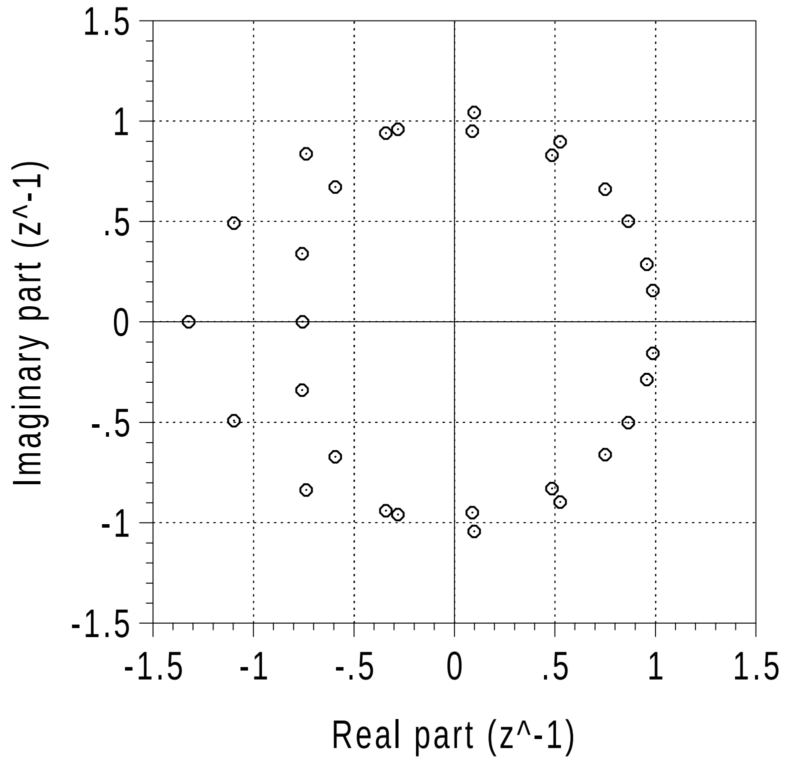
$$z_{p_{1,2}}^{-1} = 0.9588639 \pm 0.7240575j,$$
$$z_{p_3}^{-1} = 1.511628,$$
$$z_{z_{1,2}}^{-1} = -0.9049098 \pm 0.1414979j,$$
$$z_{z_3}^{-1} = -1.192327.$$

30th-order filter - accuracy compared using MATLAB and CIA



$a_4 = a_{26} =$ -0.00559342349260
$a_5 = a_{25} =$ 0.00624262031546
$a_6 = a_{24} =$ -0.00684884769613
$a_7 = a_{23} =$ 0.00897910496988
$a_8 = a_{22} =$ -0.00897865368875
$a_9 = a_{21} =$ 0.01750107253210

$a_{10} = a_{20} =$ -0.00695363594317
$a_{11} = a_{19} =$ 0.03977449895168
$a_{12} = a_{18} =$ -0.06465598090179
$a_{13} = a_{17} =$ 0.08524095225662
$a_{14} = a_{16} =$ -0.13129215646492
$a_{15} =$ 0.19514096846301



The filter has 28 complex (14 pairs) and 2 real zeros as illustrated in Fig. If compare all numerals in MATLAB and CIA output files, we obtain: 2 pairs have equal 5 valid numerals, 3 pairs have equal 6 valid numerals, and 2 real zeros and 9 pairs have equal 7 or more valid numerals.

Estimating a Frequency of Distributed Microwave Oscillator

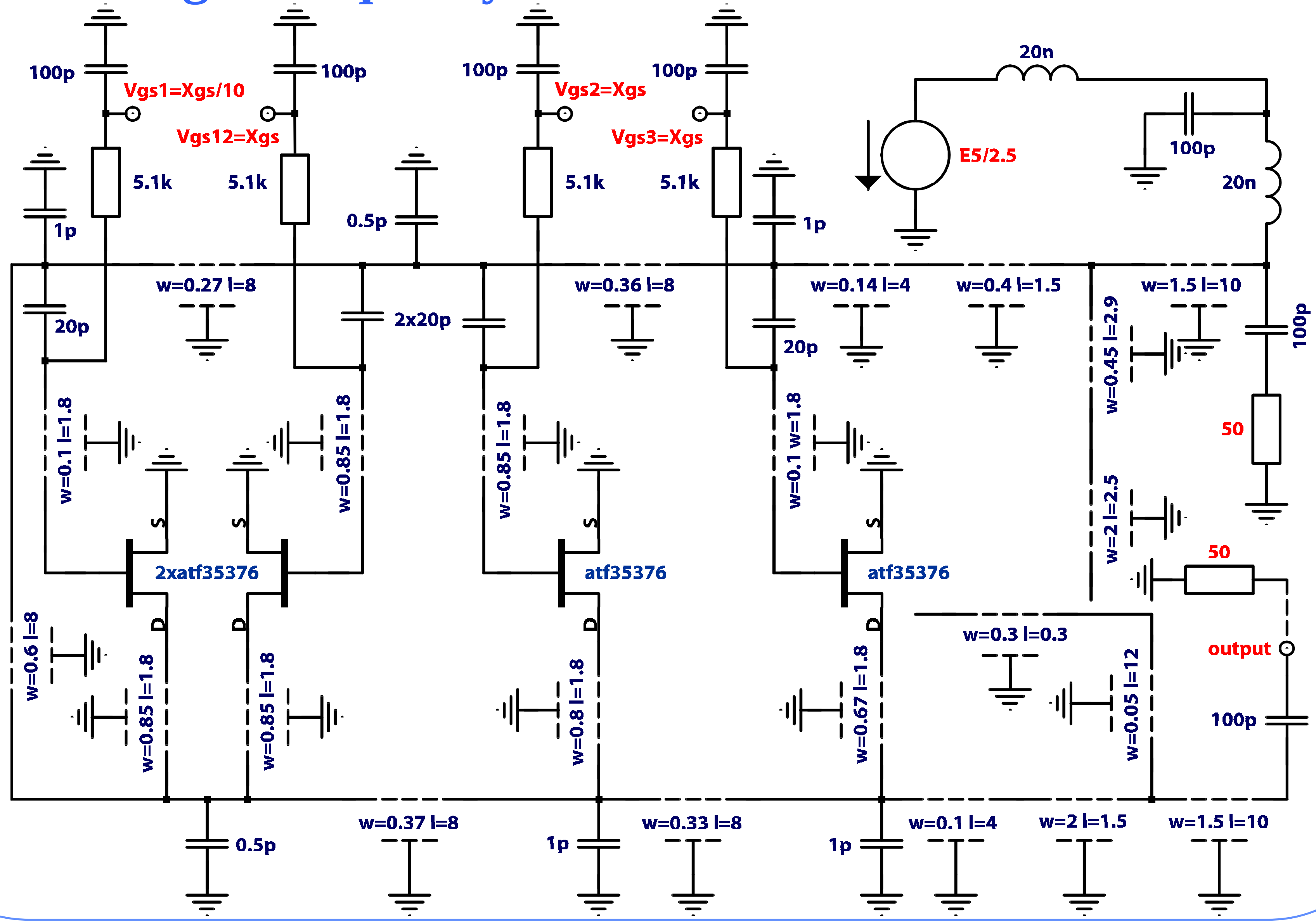
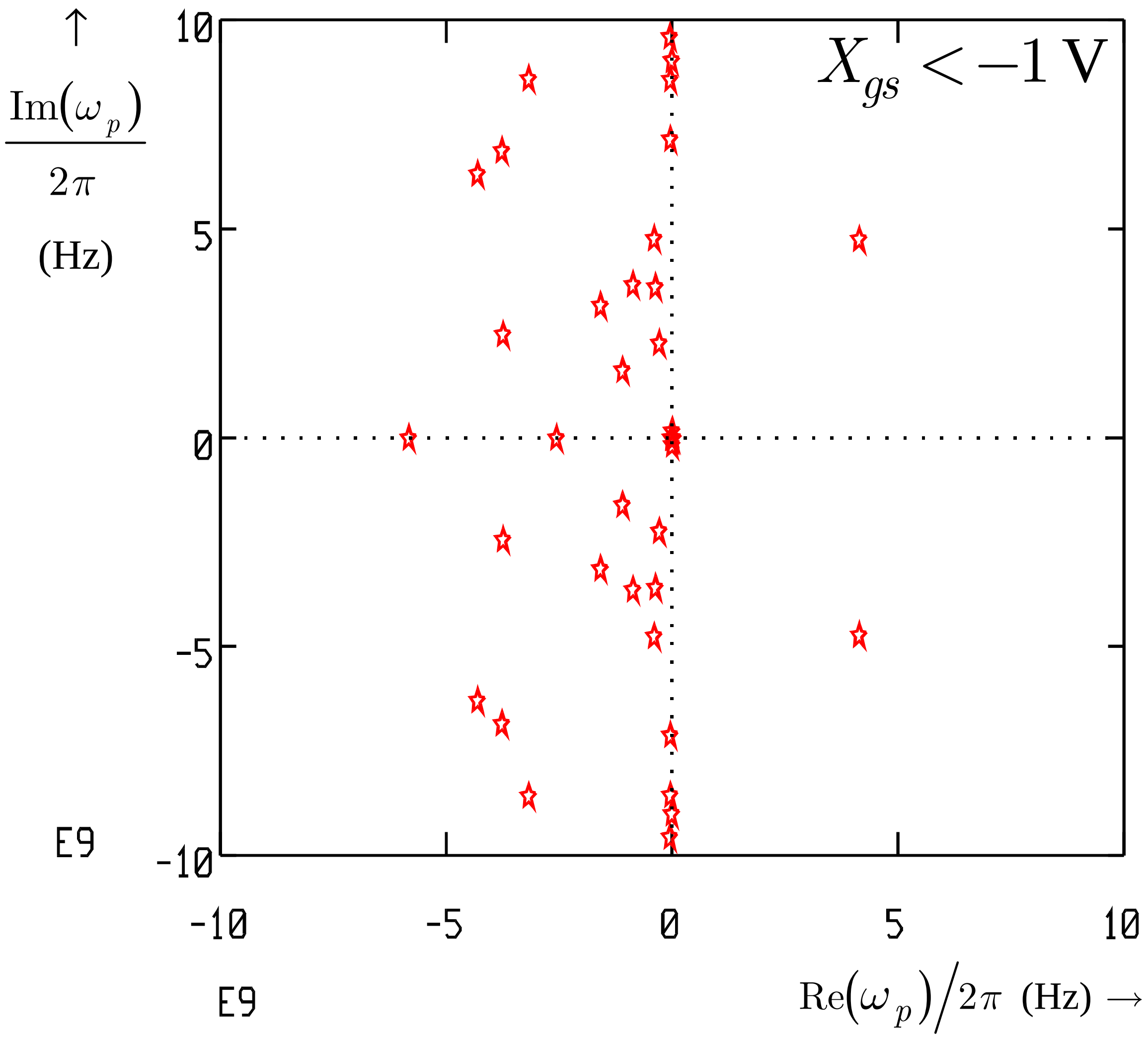
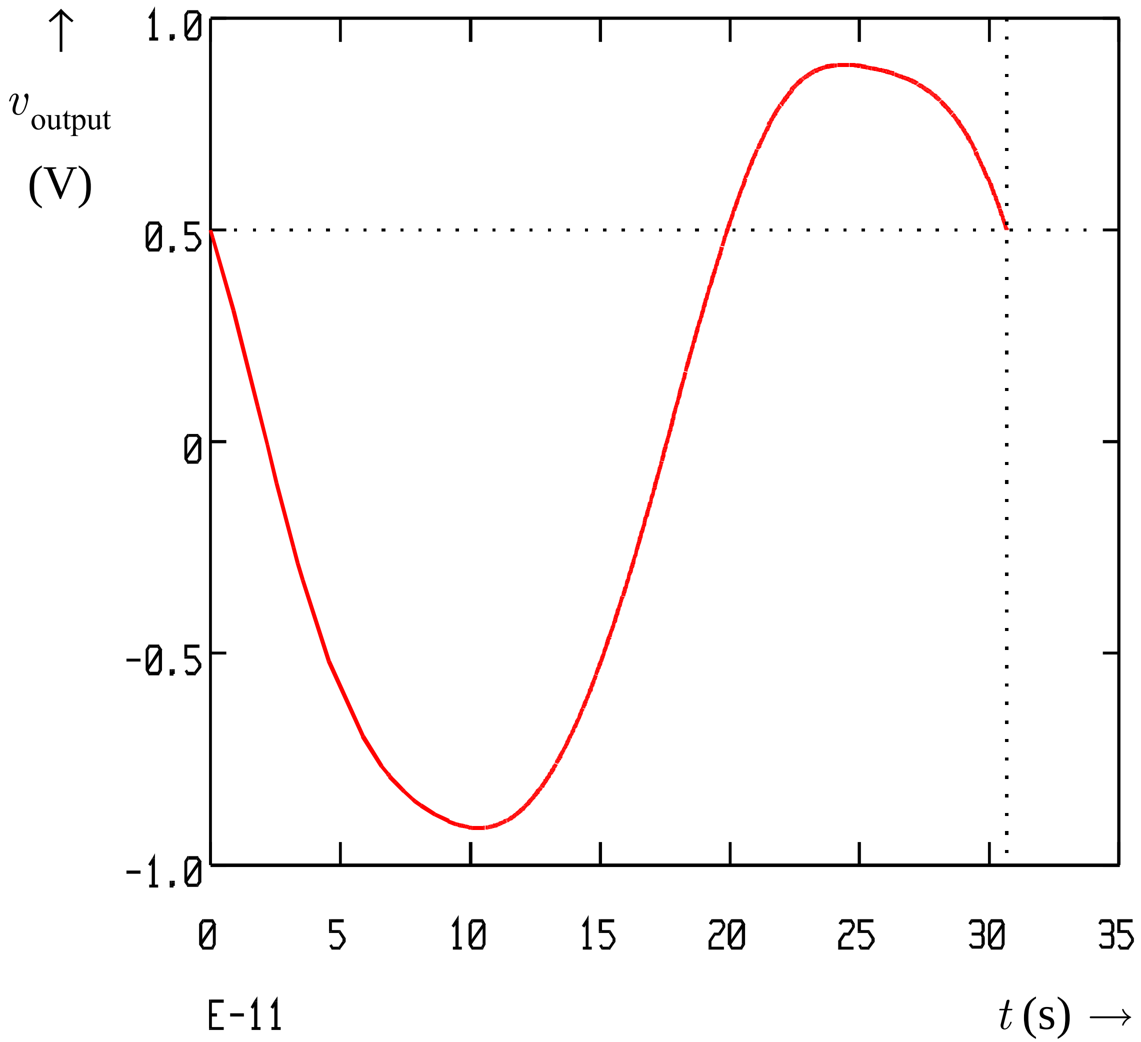


Diagram of poles of distributed oscillator:



Steady-state of the oscillator $(\Theta = 0.3068 \text{ ns})$:



Accuracy check by means of comparising poles-zeros and steady-state:

X_{gs} (V)	$f_o^{(\mathrm{ss})}$ (GHz)	$f_o^{(\mathrm{pz})}$ (GHz)	$\left(f_o^{(\mathrm{pz})} - f_o^{(\mathrm{ss})}\right) \Big/ f_o^{(\mathrm{ss})}$ (%)
-2.5	3.2733	3.1916	-2.50
-2.0	3.2916	3.3398	+1.46
-1.5	3.3670	3.2582	-3.23
-1.4	3.2308	3.1835	-1.46
-1.3	3.0693	3.3748	+9.96

Conclusion

- The poles-zeros analysis has presented as a useful tool that is not present in the PSPICE.
- Two ways of algorithm's implementation assessed: the first using full pivoting for accurate computations and the second using sparse coding for large-scale circuit analysis.
- An accuracy of sparse algorithms enhanced.
- Various abilities of the algorithms have been illustrated by solving analog and digital circuits in detailed way; especially the accuracy has been evaluated.