

Laplaceova transformace

Základní obrazy:

$$\mathrm{e}^{at} \stackrel{\wedge}{=} \frac{1}{p-a}, \quad p > a \quad (a \in \mathbb{R})$$

$$\cos \omega t \stackrel{\wedge}{=} \frac{p}{p^2 + \omega^2}, \quad p > 0 \quad (\omega \in \mathbb{R})$$

$$\sin \omega t \stackrel{\wedge}{=} \frac{\omega}{p^2 + \omega^2}, \quad p > 0 \quad (\omega \in \mathbb{R})$$

Další obrazy:

$$1 \stackrel{\wedge}{=} \frac{1}{p}, \quad p > 0$$

$$t \stackrel{\wedge}{=} \frac{1}{p^2}, \quad p > 0$$

$$t^2 \stackrel{\wedge}{=} \frac{2}{p^3}, \quad p > 0$$

\vdots

$$t^n \stackrel{\wedge}{=} \frac{n!}{p^{n+1}}, \quad p > 0 \quad (n \in \mathbb{N})$$

Přepisy pro vzory:

$$\frac{1}{(p-a)^n} \stackrel{\wedge}{=} \mathrm{e}^{at} \frac{t^{n-1}}{(n-1)!} \quad (a \in \mathbb{R}, n \in \mathbb{N})$$

$$\frac{p-a}{(p-a)^2 + \omega^2} \stackrel{\wedge}{=} \mathrm{e}^{at} \cos \omega t \quad (a, \omega \in \mathbb{R})$$

$$\frac{\omega}{(p-a)^2 + \omega^2} \stackrel{\wedge}{=} \mathrm{e}^{at} \sin \omega t \quad (a, \omega \in \mathbb{R})$$

Základní vzorce:

$$a \cdot f(t) + b \cdot g(t) \stackrel{\wedge}{=} a \cdot F(p) + b \cdot G(p), \quad (p > \max\{p_f, p_g\}) \quad (\text{linearita})$$

$$t \cdot f(t) \stackrel{\wedge}{=} -F'(p), \quad p > p_f \quad (\text{derivace obrazu})$$

$$\frac{f(t)}{t} \stackrel{\wedge}{=} \int_p^{+\infty} F(q) \mathrm{d}q, \quad p > p_f \quad (\text{integrál obrazu})$$

$$\mathrm{e}^{at} \cdot f(t) \stackrel{\wedge}{=} F(p-a), \quad p > p_f + a \quad (\text{posun v obrazu})$$

$$f(at) \stackrel{\wedge}{=} \frac{1}{a} F\left(\frac{p}{a}\right), \quad p > a \cdot p_f \quad (a > 0, \text{změna měřítka})$$

$$f'(t) \stackrel{\wedge}{=} p \cdot F(p) - f(0+), \quad (p > \max\{0, p_{f'}\}) \quad (\text{obraz derivace})$$

$$\int_0^t f(u) \mathrm{d}u \stackrel{\wedge}{=} \frac{F(p)}{p}, \quad (p > \max\{0, p_f\}) \quad (\text{obraz integrálu})$$

$$\int_0^t f(u) \cdot g(t-u) \mathrm{d}u \stackrel{\wedge}{=} F(p) \cdot G(p), \quad (p > \max\{p_f, p_g\}) \quad (\text{obraz konvoluce})$$

$$f(t) \cdot \mathrm{H}(t-a) \stackrel{\wedge}{=} \mathrm{e}^{-ap} \mathcal{L}\{f(t+a)\}, \quad p > p_f \quad (a \geq 0, \text{translace})$$

$$f(t-a) \cdot \mathrm{H}(t-a) \stackrel{\wedge}{=} \mathrm{e}^{-ap} F(p), \quad p > p_f \quad (a \geq 0, \text{translace})$$

$$f(t) \stackrel{\wedge}{=} \frac{F_T(p)}{1 - \mathrm{e}^{-Tp}}, \quad (p > 0) \quad (\text{obraz periodické funkce})$$

Další vzorce:

$$f''(t) \stackrel{\wedge}{=} p^2 F(p) - p f(0+) - f'(0+)$$

\vdots

$$f^{(n)}(t) \stackrel{\wedge}{=} p^n F(p) - p^{n-1} f(0+) - \dots - f^{(n-1)}(0+)$$